RP-003-1015003 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

February – 2019

Mathematics : MATH-07(A)

(Boolean Algebra and Complex Analysis-1)

(New Course)

Faculty Code : 003
Subject Code : 1015003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.
(2) Figure to the right indicate full marks of the questions.

1 (A) Answer the following questions in short : **4**

(1) Define : Partial order relation.

(2) If $R = \{(1,1),(1,2),(1,3)\}$ is a relation on $A = \{1,2,3\}$ then find R^{-1}.

(3) If (S_6, D) is a lattice then cub $\{2,3\}$ = ________.

(4) For the Poset $(S_{30}, D), 2^{'}$ = ________.

(B) Attempt any **one** out of **two** : **2**

(1) Consider the relation $R = \{(i, j) / |i - j| = 2\}$ on $\{1,2,3,4,5,6\}$ IS R transitive ?

(2) Define : Meet and join.

(C) Attempt any **one** out of **two** : **3**

(1) Prove that $R = \{x, y) / x, y \in z,\ x - y$ is divided by 5$\}$ is an equivalence relation.

(2) If $(L, *, \oplus, 0, 1)$ is a bounded lattice then prove that

(i) $a * 1 = a$ (ii) $a \oplus 1 = 1$

(D) Attempt any **one** out of **two** : **5**

(1) Prove that every chain is a distributive lattice.

(2) Prove that direct product of two lattice is also a lattice.

2 (A) Answer the following questions in short : 4

(1) Define Atom in Boolean algebra.

(2) Sum of all minterms of n variables is ________.

(3) For Boolean algebra $a \oplus 1$ = ________.

(4) Define sub Boolean algebra.

(B) Attempt any **one** out of **two** : 2

(1) Find the atoms of boolean algebra $(S_{30}, *, \oplus, ', 0, 1)$.

(2) Define : Boolean Homomorphism.

(C) Attempt any **one** out of **two** : 3

(1) If a and b are distinct atoms of Boolean algebra $(B, *, \oplus, ', 0, 1)$ then prove that $a * b = 0$.

(2) Write all the minterms of three variables.

(D) Attempt any **one** out of **two** : 5

(1) State and prove D' Morgan's law for the Boolean algebra.

(2) If $(B, *, \oplus, ', 0, 1)$ is a Boolean algebra then prove that any $x_1, x_2 \in B$ (1) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$

(2) $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$.

3 (A) Answer the following questions in short : 4

(1) Define analytic function.

(2) State Laplace equation in Cartesian form.

(3) If $f(z) = u(r, \theta) + iv(r, \theta)$ is analytic function then $f'(z)$ = __________.

(4) State Cauchy-Riemann condition in cartesian form.

(B) Attempt any **one** out of **two** : 2

(1) Show that $u = \log r$ is harmonic.

(2) If $f(z) = e^x(\cos y - i \sin y)$ then prove that $f'(z) = -f(Z)$.

(C) Attempt any **one** out of **two** : 3

(1) If $w = x^2 + ayx + by^2 + i(cx^2 + dxy + y^2)$ is analytic then find value of a, b, c, d.

(2) Show that $\dfrac{y}{x^2 + y^2}$ is harmonic function and find its conjugate harmonic function.

(D) Attempt any **one** out of **two** : **5**

(1) Prove that the analytic function of constant modulus is also constant in its domain.

(2) Prove that $f(z) = \begin{cases} \dfrac{(\overline{z})^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$

Satisfied Cauchy-Riemann conditions at origin however $f(z)$ is not analytic function at origin.

4 (A) Answer the following questions in short : **4**

(1) Define : closed curve.

(2) If $c : z - z_0 = r_0 e^{i\theta}$ then $\int_c \frac{dz}{z - z_0}$ ________..

(3) If $c : |z| = 1$ then $\int_c \frac{z}{2z-1} dz$ = ________.

(4) If $c : |z| = 1$ then $\int_c \frac{dz}{z-2}$ = ________.

(B) Attempt any **one** out of **two** : **2**

(1) Find $\int_0^{2+i} (\overline{z})^2 dz$.

(2) Find $\int_C \frac{dz}{z^2 + 4}$ where $c : |z - i| = 2$.

(C) Attempt any **one** out of **two** : **3**

(1) Prove that $\left| \int_c \frac{1}{z^4} dz \right| \leq 4\sqrt{2}$, where c is the line sigment joining $z = i$ to $z = 1$.

(2) State and prove Cauchy's fundamental theorem.

(D) Attempt any **one** out of **two** : **5**

(1) In usual notation prove that

$$\left| \int_a^b f(z) dz \right| \leq \int_a^b |f(z)| \, dz.$$

(2) Find $\int_c (3z+1)dz$, where c is a square joining points $z=0, z=1, z=i$ and $z=1+i$.

5 (A) Answer the following questions in short : **4**

(1) State the Cauchy-integral formula for first derivative.

(2) State Liouville's theorem.

(3) If $c : |z| = 1$ then $\int_c \frac{\cosh z}{z^4} dz$ = __________.

(4) If $c : |z| = 1$ then $\int_c \frac{e^z}{z^3} dz$ = __________.

(B) Attempt any **one** out of **two** : **2**

(1) Evaluate $\int_c \frac{5z^2 - 7z + 3}{(z-1)^2} dz$ where $c : |z| = 2$.

(2) State maximum modulus principle.

(C) Attempt any **one** out of **two** : **3**

(1) State and prove Cauchy inequality.

(2) Evaluate $\int_c \frac{\sin^6 z}{\left(z - \frac{\pi}{6}\right)^3} dz$ where $c : |z| = 1$.

(D) Attempt any **one** out of **two** : **5**

(1) State and prove fundamental theorem of Algebra.

(2) Let c be any simple closed contour described in the Positive sense in the z-plane and $g(w) = \int_c \frac{z^3 + 2z}{(z-w)^3} dz$ show that $g(w) = 6\pi i w$ when w is inside c and what the value of $g(w)$ when w is outside c.